Insurance for Improving User Satisfaction Level

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Appendix (available online via http://link.springer.com)

Appendix A

Proof of Proposition 1: In this appendix, a proof is provided for Proposition 1, for $n = 2$ and $m = 1$. The proof for any other values of n and m is similar. Assume that two users with similar characteristics but different levels of risk aversion request insurance, and that User 1 is more risk-averse than User 2. Let $u_i(w - X)$ denote the utility of the primary service for user i (i = 1, 2). Here X is a **random variable** that represents magnitude of loss. When a user uses the primary service under normal conditions, X is zero and the utility is $u(w_0)$; however, when the service provider fails to serve the user according to the SLA, X will be a non-zero value and the utility of the user decreases to $u(w_0 - X)$. Since the two users are similar, it is assumed that $u_1(w_0 - X) = u_2(w_0 - X)$. In the insurance literature (see Chapter 2 of Zweifel and Eisen (2012) for a detailed discussion), it has been shown that a risk-averse user is indifferent to the choice between a situation with utility $u(w_0 - EX - \rho_i)$ and a risky situation with utility $u(w_0 - X)$. Thus, Equation (A-1) can be written as follows:

$$u_i(w_0 - EX - \rho_i) = E(u_i(w_0 - X)) \quad i = 1,2 \tag{A-1}$$

For a risk-neutral user, ρ_i is zero; however, for a risk-averse user, ρ_i is positive. Eq. (A-2) gives another form of Eq. (A-1):

$$w_0 - EX = u_i^{-1}(E(u_i(w_0 - X))) + \rho_i \qquad i = 1,2 \tag{A-2}$$

Eqs. (A-3) and (A-4) can be inferred from Eq. (A-2):

$$u_1^{-1}(E(u_1(w_0 - X))) + \rho_1 = u_2^{-1}(E(u_2(w_0 - X))) + \rho_2 \tag{A-3}$$

$$\rho_1 - \rho_2 = u_2^{-1}(E(u_2(w_0 - X))) - u_1^{-1}(E(u_1(w_0 - X))) \tag{A-4}$$

The risk aversion of the user can be calculated using $-u''/u'$. Since User 1 is more risk-averse than User 2, then Eq. (A-5) holds:

$$u_1''(w)/u_1'(w) - u_2''(w)/u_2'(w) = (u_1''(w)u_2'(w) - u_2''(w)u_1'(w))/(u_1'(w)u_2'(w)) < 0 \tag{A-5}$$

Eq. (A-6) illustrates an integration rule:

$$\int (u_1''(w)u_2'(w) - u_2''(w)u_1'(w))/(u_1'(w)u_2'(w))dw = \log(u_1'(w)/u_2'(w)) + c \tag{A-6}$$

The RHS of Eq. (A-6) decreases, since $(u_1''(w)/u_1'(w) - u_2''(w)/u_2'(w))$ is negative according to Eq.(A-5). This means that $u_1'(w)/u_2'(w)$ decrease as well. Therefore, Eq. (A-7) can be concluded:

$$d(u_1'(w)/u_2'(w))/dw < 0 \tag{A-7}$$

Moreover, since $u_i'(w) > 0$ (for $i = 1,2$), then Eq. (A-8) holds:

$$u_1'(w)/u_2'(w) > 0 \tag{A-8}$$

Now let $z = u_2(w)$. Then $u_1'(w)/u_2'(w)$ can be rewritten as $u_1'(u_2^{-1}(z))/u_2'(u_2^{-1}(z))$. In addition, Eq. (A-9) can be written as follows:

$$\int u_1'(u_2^{-1}(z))/u_2'(u_2^{-1}(z))dz = u_1(u_2^{-1}(z)) + c \tag{A-9}$$

The first derivative of $u_1(u_2^{-1}(z))$ is positive, as illustrated by Eq. (A-8). Since $dz/dw>0$, therefore $u_1'(u_2^{-1}(z))/u_2'(u_2^{-1}(z))$ is decreasing in accordance with Eq. (A-7). This means that $u_1(u_2^{-1}(z))$ is concave, and using Jensen's inequality, Eq. (A-10) can be written as follows:

$$E(u_1(u_2^{-1}(z))) \le u_1(u_2^{-1}(E(z))) \tag{A-10}$$

Substituting $z_0 = u_2(w_0 - X)$ or $w_0 - X = u_2^{-1}(z_0)$ into Eq. (A-3) gives Eq. (A-11):

$$\rho_1 - \rho_2 = u_2^{-1}(E(z_0)) - u_1^{-1}(E(u_1(u_2^{-1}(z_0)))) \tag{A-11}$$

Since $u_1' > 0$ and from Eq.(A-10), Eq. (A-12) can be concluded:

$$u_2^{-1}(E(z_0)) \ge u_1^{-1}(E(u_1(u_2^{-1}(z_0)))) \Rightarrow \rho_1 \ge \rho_2 \tag{A-12}$$

It has been assumed that $u_1(w_0 - X) = u_2(w_0 - X)$ for each value of X. Thus, $E(u_1(w_0 - X)) = E(u_2(w_0 - X))$, and it is then straightforward that $u_1(w_0 - EX - \rho_1) = u_2(w_0 - EX - \rho_2)$. Since $\rho_1 \ge \rho_2$ and $u_1' > 0$, Eq. (A-13) holds:

$$u_1(w_0 - EX - \rho_2) \ge u_2(w_0 - EX - \rho_2) \tag{A-13}$$

Provided that $u_1(w_0) = u_2(w_0)$, $u_1(w_0 - X) = u_2(w_0 - X)$, $u_i' > 0$ $(i = 1,2)$, $u_i'' < 0$ $(i = 1,2)$ and from Eq. (A-13), it can be shown that

$$u_1(z) \ge u_2(z) \quad \forall z \in [w_0 - X, w_0] \tag{A-14}$$

Since the insurer can provide insurance to only one user, one of the users must be rejected. The utility of the rejected user will be $E(u_i(w_0 - X))$. If User i receives insurance, his/her utility will be $u_i(w_0 - pr)$ under normal conditions, and $u_i(w_0 - X + X_0 - pr)$ in the case of an SLA violation. Here, X_0 is the indemnity, and in full coverage insurance, this is equal to X. For partial coverage, $X_0 < X$. The term pr represents the insurance premium, and is smaller than X_0. Therefore, both $(w_0 - pr)$ and $(w_0 - X + X_0 - pr)$ are within the interval $[w_0 - X, w_0]$. For all values within $[w_0 - X, w_0]$ and for any combination of these, $u_1 \ge u_2$ holds. In other words, if an insurer rejects User 2, the service provider can reach a higher USL than in the case where the insurer rejects User 1. A rational service provider therefore assigns a higher priority to User 1, who is more risk averse. Q.E.D.

Appendix B

Proof of Lemma 1: If the favorites are not cycle-free, there is therefore a wrap-around sequence of brokers $b_1, b_2, ..., b_k$, such that each broker b_i prefers b_{i+1} to b_{i-1}. Assume b_i to be an iBroker. Since iBrokers arrange their favorites lists based on the priority of users, these lists of favorites are similar. This means that if iBroker b_i prefers uBroker b_{i-1} to uBroker b_{i+1}, every other iBroker has the same preference. Since this is a wrap-around sequence and preferences are transitive, after the first iteration of the sequence, it can be obtained that iBroker b_i prefers uBroker b_{i+1} to uBroker b_{i-1}. This result contradicts the definition of the sequence. The existence of such a sequence of brokers is therefore not possible, and the favorites are cycle-free. Q.E.D.

Appendix C

Proof of Theorem 1: Start with an arbitrary uBroker from the set of brokers, for example b_1, and form a sequence $b_1, b_2, ...$, in which b_{i+1} is the broker most preferred by broker b_i of the other existing brokers in the set of the opposite type. Since the number of brokers is limited, the sequence therefore has a cycle. According to Lemma 1, the obtained favorites are cycle-free, and therefore the cycle is of length 2. This cycle contains two brokers which have the highest preference for each other. By eliminating these two brokers from the set of brokers, a similar process with fewer brokers can be carried out at the next iteration. This process then resumes until the last iteration. Q.E.D.

Appendix D

Proof of Theorem 2: Assume that the current strategy profile of the players in uGame and iGame is equivalent point to the outcome of the pairing algorithm. All the iBrokers have identical favorites lists; hence, they have identical rankings of uBrokers. Let uB_i denote the uBroker in the *i*th place of this ranking, and let iB_i denote the iBroker paired to uB_i by the pairing algorithm.

In uGame, if uB_i deviates unilaterally and selects iB_k ($k \neq i$), then for the case of $r_{uB_i}(iB_k) > r_{uB_i}(iB_i)$, this deviation is unprofitable. However, if $r_{uB_i}(iB_k) < r_{uB_i}(iB_i)$, then iB_k has paired with a uB_k such that uB_k has a higher priority than uB_i, and every iBroker prefers uB_k to uB_i. Therefore, iB_k refuses uB_i and according to the definition of uGame, $u_{uB_i}(s_{uB_i} = iB_k) = 0$. Consequently, for the case of $r_{uB_i}(iB_k) < r_{uB_i}(iB_i)$, this deviation is also unprofitable. This means that the outcome of the pairing algorithm is a pure Nash equilibrium of uGame.

For iGame, all iBrokers prefer uBroker uB_i to uB_{i+1}. This means that the deviation of iB_1 is unprofitable. The deviation of iB_2 will be profitable if uB_1 prefers iB_2 to iB_1. However, this cannot hold, since based on the pairing algorithm, uB_1 and iB_1 are two brokers which have the highest preference for each other. Thus, the deviation of iB_2 is also unprofitable. It can be concluded in the same way that for the rest of the iBrokers, deviation is unprofitable, and the outcome of the pairing algorithm will also be a pure Nash Equilibrium of iGame. Q.E.D.

Appendix E

Proof of Theorem 3: Assume that the PNE of iGame and uGame is not unique and that there is another Nash equilibrium $\underline{NE}$. Let $\overline{NE}$ denote the PNE, which is equivalent to the outcome of the pairing algorithm; the definitions of uB_i and iB_i are similar to those in Theorem 2. Let $s_B(N)$ denote the strategy of broker B in a strategy profile N of the game.

For uGame, if $s_{uB_1}(\underline{NE}) \neq iB_1$ then the deviation of uB_1 to iB_1 will be profitable, and $\underline{NE}$ is not a Nash point. For the case of $s_{uB_1}(\underline{NE}) = iB_1$, we have $s_{uB_1}(\underline{NE}) = s_{uB_1}(\overline{NE})$. It can be shown that for all uBrokers, the condition of $s_{uB_1}(\underline{NE}) = s_{uB_1}(\overline{NE})$ is required for $\underline{NE}$ to be a Nash point. Thus, $\overline{NE}$ is unique. The proof for iGame can be carried out in a similar way. Q.E.D.